

An Examination of the Updated Empirical Data in Support of the Shell Model

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Abstract

This paper is a re-examination of the experimental data, as is currently known in 2020, in support of the shell model and its concepts. The shell model of the nuclear force was proposed in 1955, and it has since been accepted as the fundamental model of the nuclear force. The shell model was based on the experimentally-known data at that time, supporting the concept of nuclear shells--a concept similar to the quantum atomic shells formed by electrons. Most textbooks, even the current ones, present this experimental data from the 1950's when discussing the validity of the shell model. A large amount of nuclear data has since been collected over the past 60 years, and a re-examination of the experimental data in support of the shell model is overdue.

Introduction

The nuclear shell model [1, 2] is constructed from the concept of the electronic shell structure of atoms. When the nuclear shell model was first proposed, there were immediate objections to the it, since it is based on a centrally-located force, such as the gravitational force on the surface of a planet. However, the nuclear force is not a centrally-located force, an implication indicated by the nuclear binding curve. As A , the number of nucleons increases, centrally-located force would have a parabolically increasing curve for binding energy versus A . Experimentally, this is not seen. Rather, the nuclear force is commonly called a saturated force, in which the (binding energy/ A) vs A is relatively flat, or "saturated".

The shell model claims to explain several behaviors of the nuclear force, which will be discussed in detail in this paper. However, it can only explain these behaviors if there are additional terms and variables incorporated into it. When employing the Schrödinger equation, these additional variables are added to the shell model to account for geometrical considerations, vibrations, rotational excitations, and pairing properties [3]. The shell model includes "magic" numbers, numbers that are found by examining empirical data and searching for any discrepancies or discontinuities. The magic numbers are when either Z or N is equal to 2, 8, 20, 28, 50, 82, and 126. The shell model attempts to correct these discrepancies by claiming that the nuclear structure is based on shells, similar to the electronic shells in an atom.

The latest and most updated databases are used to update the empirical nuclear data [4]. This updated data is used to reconstruct the diagrams, graphs, and other empirical evidence in support of the shell model. These updates include the graphs for nuclear binding energy, the number of stable isotopes, separation energies, quadrupole moments, and other empirical data would support the existence of nuclear shells.

Claim 1: There are incongruities in the binding energy per nucleon when comparing experimental data to the predictions of the semi-empirical formula.

When comparing the binding energy per nucleon to the predicted value from the semi-empirical formula, one can see a slight bump at $Z=28, 50, 82$ and at $N=28, 50, 82, \text{ and } 126$. Fig. 1a and Fig. 1b show the difference between the semi-empirical formula calculations and the actual binding energy per nucleon, versus Z and N . As can be seen, this incongruity is small. Usually, the left sides of these graphs are not shown, to hide the very large discrepancies there. These charts are generated by inserting the values of Z , N , and A into the semi-empirical equation, and then subtracting the experimental values binding energy for each nuclide.

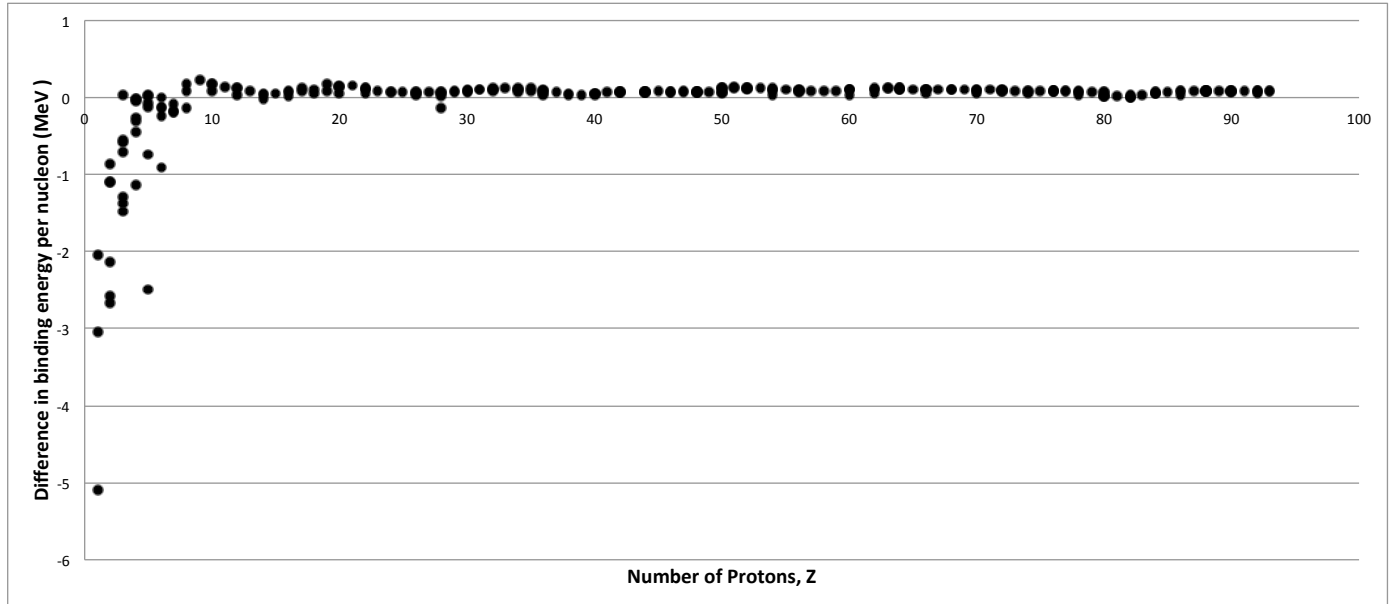


Fig. 1a Difference between semi-empirical formula and actual binding energy per nucleon, versus Z.

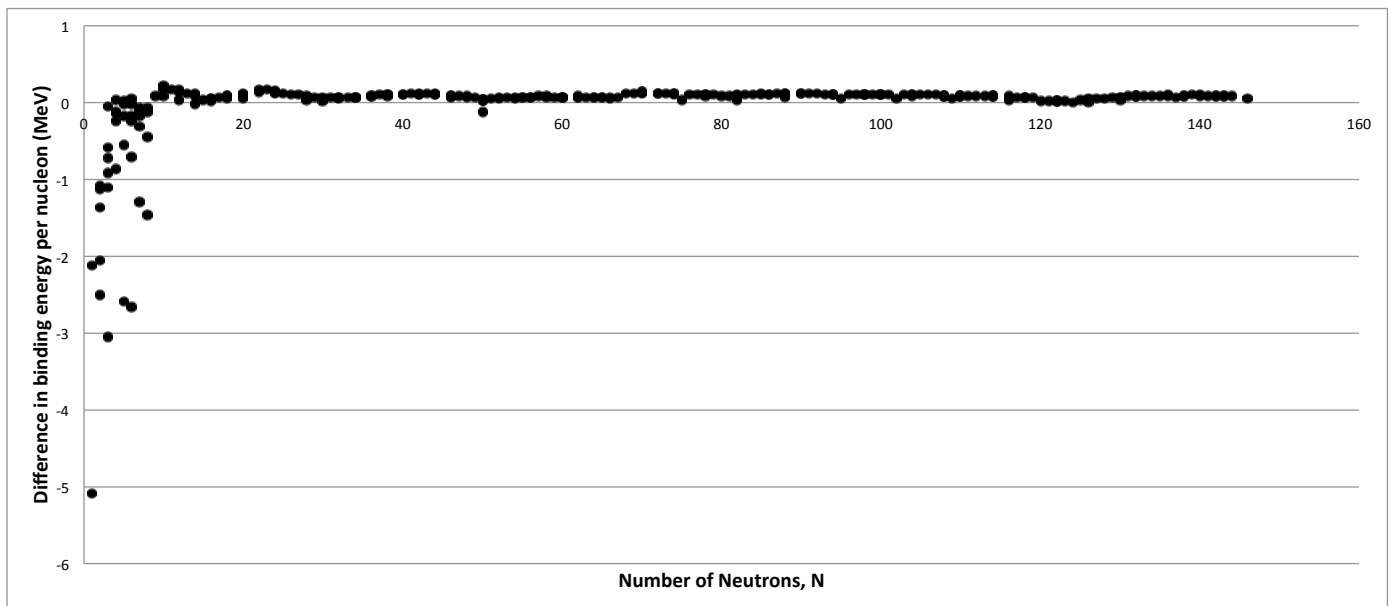


Fig. 1b Difference between semi-empirical formula and actual binding energy per nucleon, versus N.

The slight discrepancies seen at the magic numbers seem to be little more than a minor variation, especially when compared to the large discrepancies of the smaller nuclides.

Claim 2: There are more isotopes when Z is a magic number, and there are more isotones when N is a magic number.

A second claim for evidence of shells is that there are more known isotopes when Z is equal to a magic number. And similarly, there are more isotones when N is equal to a magic number. However, as seen in the Fig. 2a and Fig. 2b, there is not a significant variation in the number of isotopes or isotones for the magic numbers [4]. The updated data shows this to be an insignificant effect.

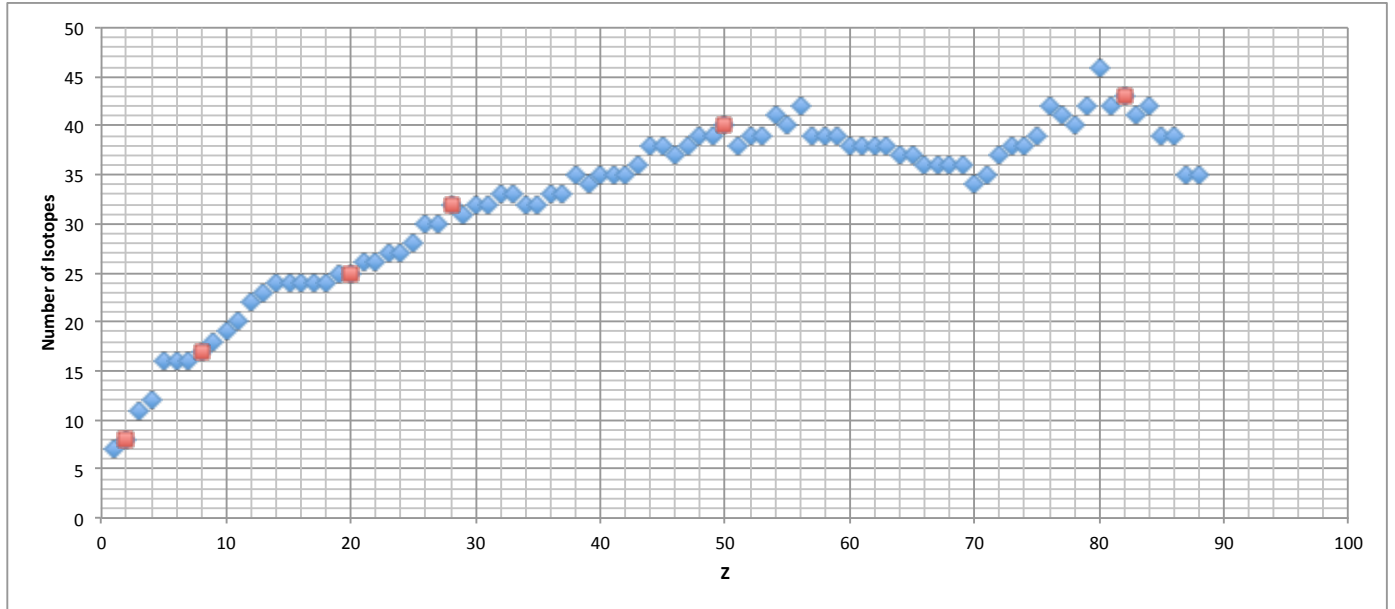


Fig. 2a The number of isotopes, versus Z . Magic numbers are highlighted in red

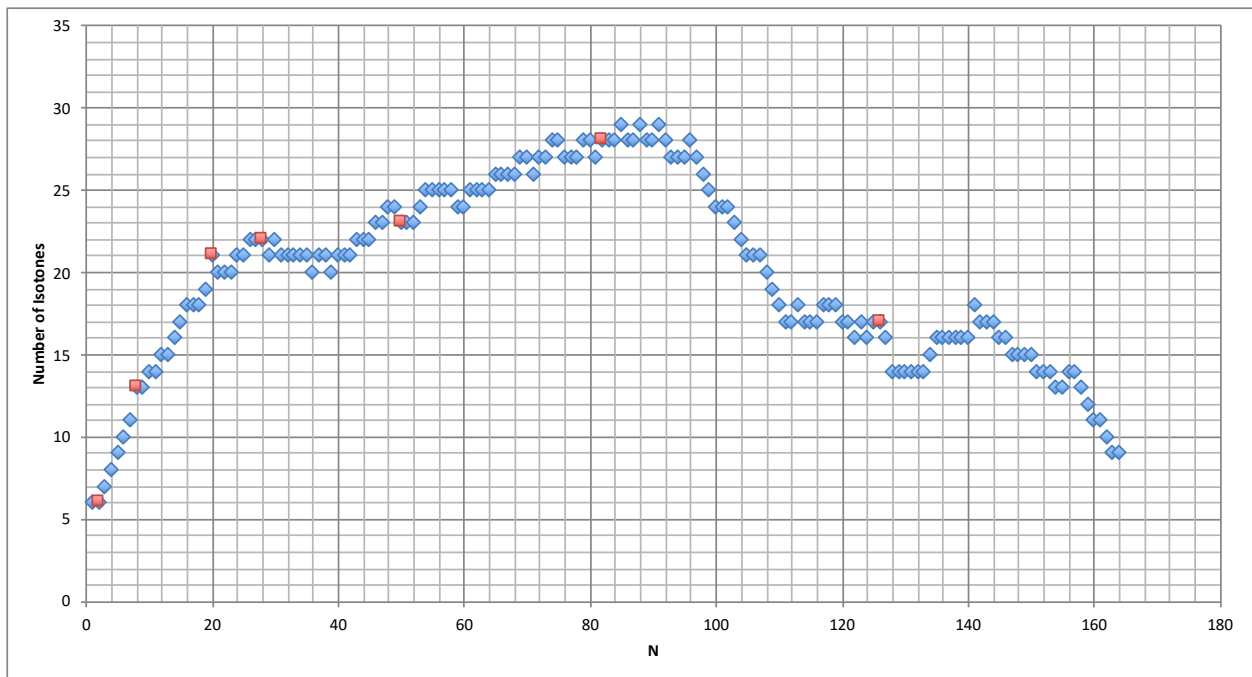


Fig. 2b The number of isotones, versus N . Magic numbers are highlighted in red

Claim 3: Nuclei with a magic number for Z or N have a higher binding energy than non-magic nuclei.

Another claim of the shell model is that nuclei with either Z or N equal to a magic number have a higher binding energy than non-magic nuclei. Figures 3a and 3b show the experimental binding energy for all the nuclides, stable and unstable. Figure 3a shows the binding energy vs. Z , and Figure 3b shows the binding energy versus N . As is seen in the figures, there is not a higher binding energy for the magic numbers. The evidence of there being higher binding energy for nuclides with magic numbers at 2, 8, 20 or 28 is not evident in either graph, and going out further to larger values of Z or N also shows no such effect in binding energy.

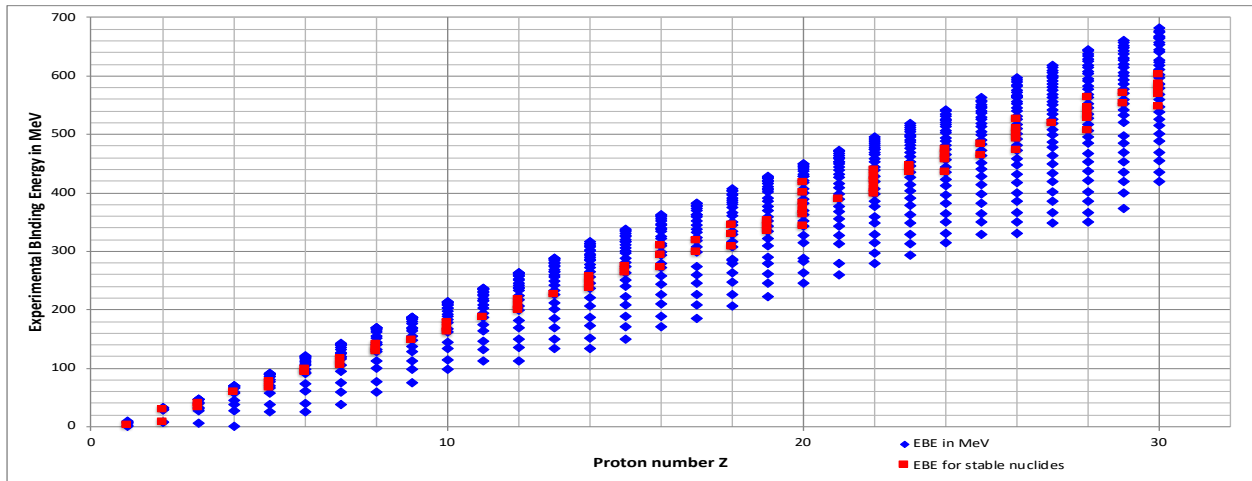


Fig. 3a Experimental binding energy per nucleon versus Z.

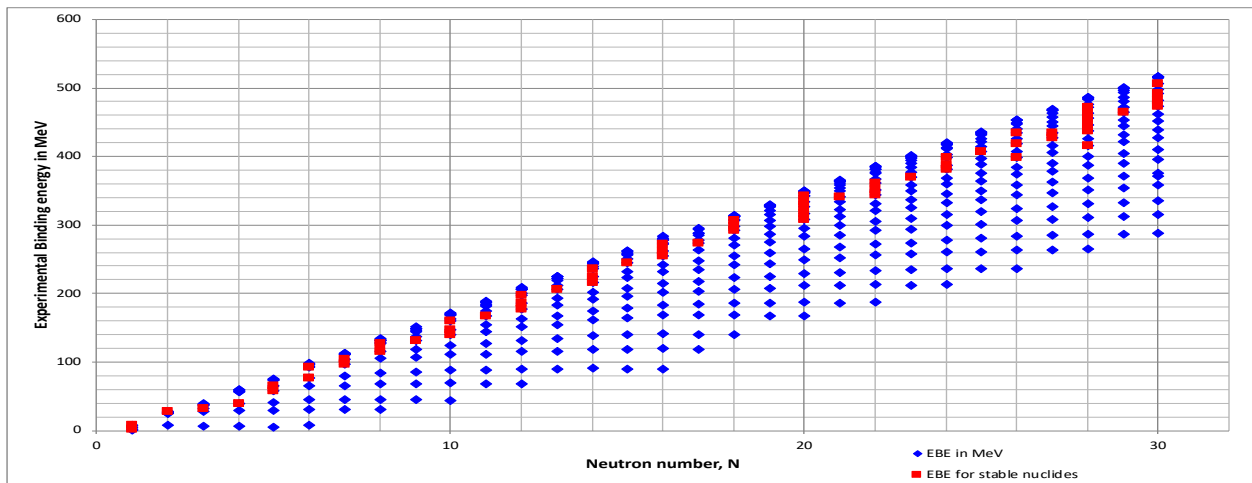


Fig. 3b Experimental binding energy per nucleon versus N.

As seen in Fig. 3a and 3b, the nuclides with a magic number of either 2, 8, 20 or 28 are not more tightly bound than the other nuclides near them. For example, all nuclides of Helium have the magic number of $Z=2$, but other than ${}^3\text{He}$ and ${}^4\text{He}$ they are not stable, nor is ${}^3\text{He}$ tightly bound. Helium 5 is extremely unstable, even though it has a magic number for the number of protons. Similarly the nuclides with $N=8$, such as ${}^{11}\text{Li}$, are not more tightly bound nor more stable than the non-magic values of N.

Another related claim of the shell model suggests that the “doubly magic” nuclides are more stable and more tightly bound, such as ${}^4\text{He}$ and ${}^{16}\text{O}$. However, ${}^4\text{He}$ and ${}^{16}\text{O}$ are not distinct in their binding energy per nucleon, when compared to other alpha-particle nuclides, such as ${}^{12}\text{C}$, ${}^{20}\text{Ne}$, ${}^{24}\text{Mg}$, ${}^{28}\text{Si}$, ${}^{32}\text{S}$, ${}^{36}\text{Ar}$, or ${}^{40}\text{Ca}$. This is shown in Fig. 4 for the Experimental Binding Energy per nucleon versus A.

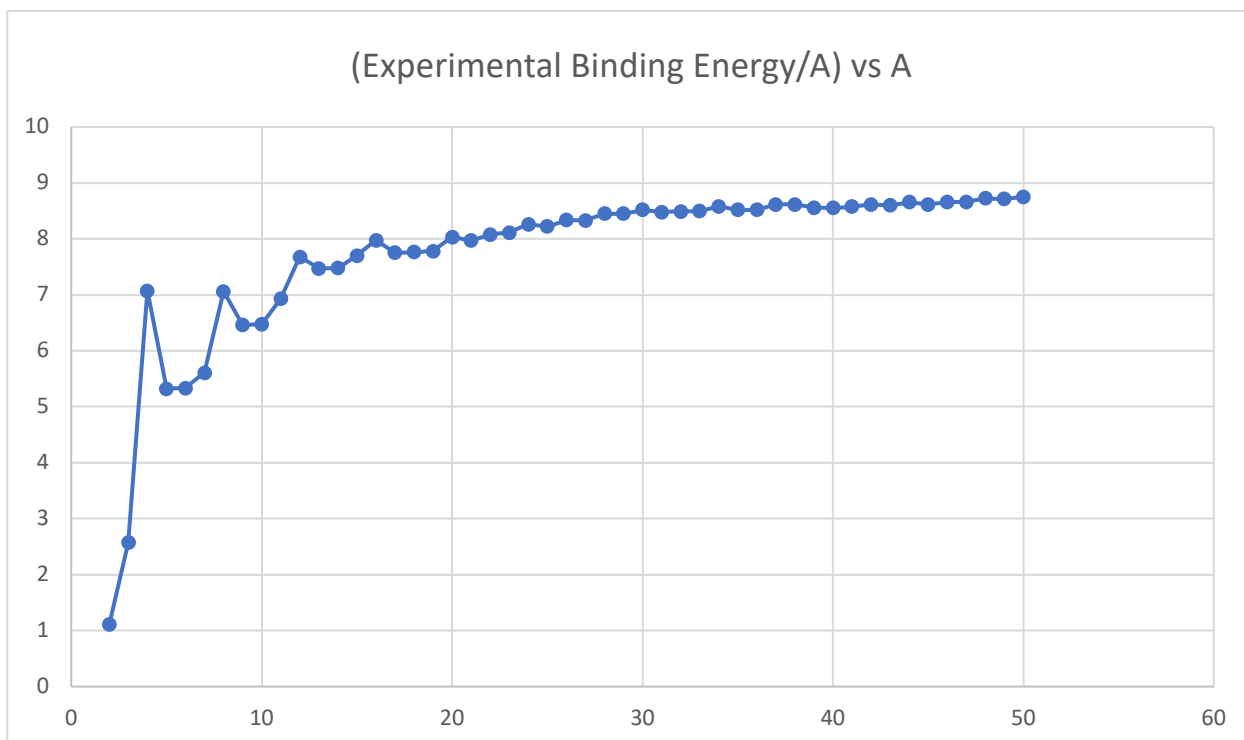


Fig. 4 Experimental binding energy per nucleon versus A for the first 50 stable nuclides.

All of the other alpha-particle nuclides (in which Z is even, and $N=Z$) have a slightly higher binding energy than the nuclides near them. Thus, this higher binding energy per A at the values of $A=4, 8, 12, 16, 20, 24, 28,$ and 32 supports the alpha-particle cluster model more than the shell model. Also, the claim that doubly magic nuclides are more stable is not a valid, as can be seen when examining all the doubly magic nuclides. There are 12 known nuclides that are doubly magic: ${}^4\text{He}, {}^{10}\text{He}, {}^{16}\text{O}, {}^{28}\text{O}, {}^{40}\text{Ca}, {}^{48}\text{Ca}, {}^{48}\text{Ni}, {}^{56}\text{Ni}, {}^{78}\text{Ni}, {}^{100}\text{Sn}, {}^{132}\text{Sn},$ and ${}^{208}\text{Pb}$. However, only five of them are stable: ${}^4\text{He}, {}^{16}\text{O}, {}^{40}\text{Ca}, {}^{48}\text{Ca},$ and ${}^{208}\text{Pb}$. The remaining 7 nuclides are unstable. Thus, being doubly magic does not endow a nuclide more stability.

Claim 4: Neutron Separation Energies show a distinct zigzag pattern associated with magic numbers, characteristic of shells.

Another claim in support of shell model numbers is the graph reproduced in Fig. 5, showing the separation energy versus neutron number N. Fig. 5 is a reproduction of a graph using data from the 1950's. The separation energy, S_n , is the energy required to remove one neutron from a nuclide. This graph in Fig. 5 attempts to emphasize the effect of magic numbers. To obtain this graph, the binding energy of the two nuclides A_N and $A_{(N-1)}$, are subtracted. Note that these binding energies go up to 1800 MeV for the larger nuclides, thus the difference between the two illustrated in this figure is a very small difference, 1 to 2 MeV, between two much larger numbers. This difference, obtained from the experimental data, is then compared to the predictions of the semi-empirical binding mass formula, and the resulting difference of the differences, is plotted as shown in Fig. 5.

The zigzag effect seen in this graph is the reason this graph is considered to support the nuclear shell model. For the shell model for electrons, the zigzag in the energy is due the electrons forming shells around the nucleus. A plot of the binding energy of the last electron in an atom versus the number of electrons displays this zigzag pattern, a pattern which shows that as the electronic shells are filled, the binding energy of the last electron exhibits a zigzag in the amount of energy required to remove it. If the nuclear shells existed within a nucleus, similar to the electronic shells around an atom, then this same zigzag effect should be seen.

Separation Energy of the last neutron, S_N
 The difference between experimental data and semi-empirical formula

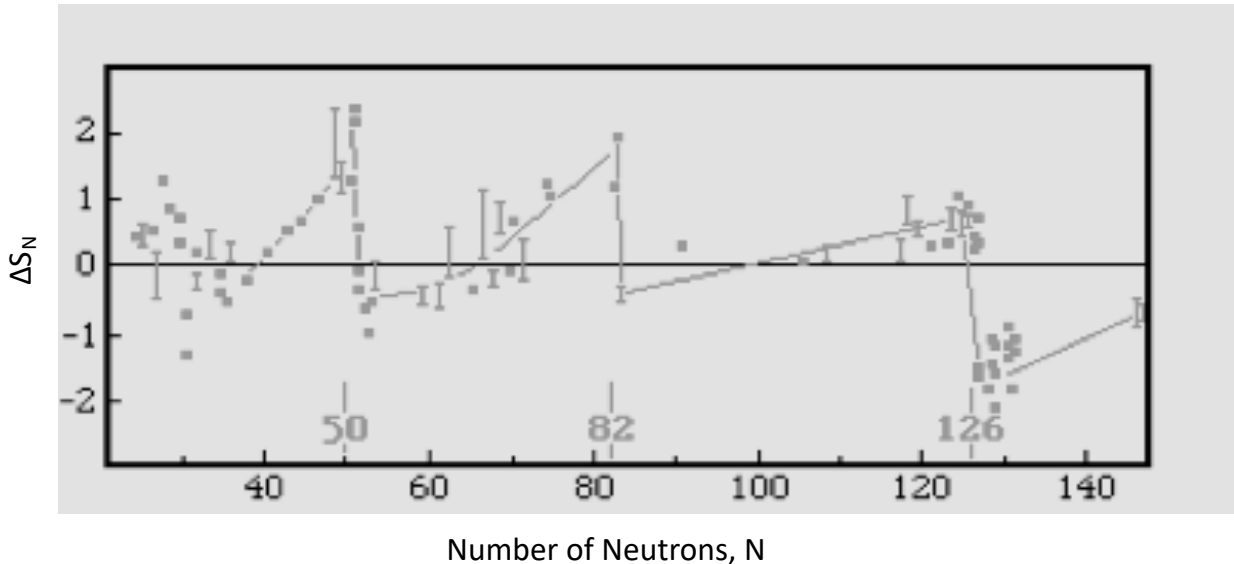


Fig. 5 A graph of the binding energy of last neutron versus N , data from the 1950's.

One obvious problem with this graph in Fig. 5 is that it does not include all the nuclides. Nor does it even include, at the very least, all the stable nuclides. There are over 3200 known nuclides that should be plotted on this graph, and yet the data shown in Fig. 5 only shows a smattering of selected points.

Also, the graph in Fig. 5 is the difference between the empirical separation energy and what is predicted by using the semi-empirical model, with no justification for taking the difference between the experimental and theoretical separation energies. The electronic shell model data for electron separation energy is simply a plot of the separation energy, not the difference between the separation energy and a theoretical separation energy. The experimental data for the separation energy should show a zigzag without having to subtract it from another model's theoretical predictions.

Another problem with this graph is that it cuts off at $N < 20$, not showing the data for the smaller nuclides. When the nuclides from 1 to 20 are included, the resulting spread is extremely large and is unrelated to magic numbers. Hence, for all these reasons, the graph in Fig. 5 is not a true representation of S_n versus N .

Fig. 6 shows all the actual separation energy, S_n , of all the known nuclides. (All data for S_n has been extracted from reference [4].) As can be seen by comparing Fig. 5 to Fig. 6, there is much difference between what is purported to be proof a shell-like structure in a nucleus in Fig. 5, and what is actually seen in the updated empirical data for neutron separation energy.

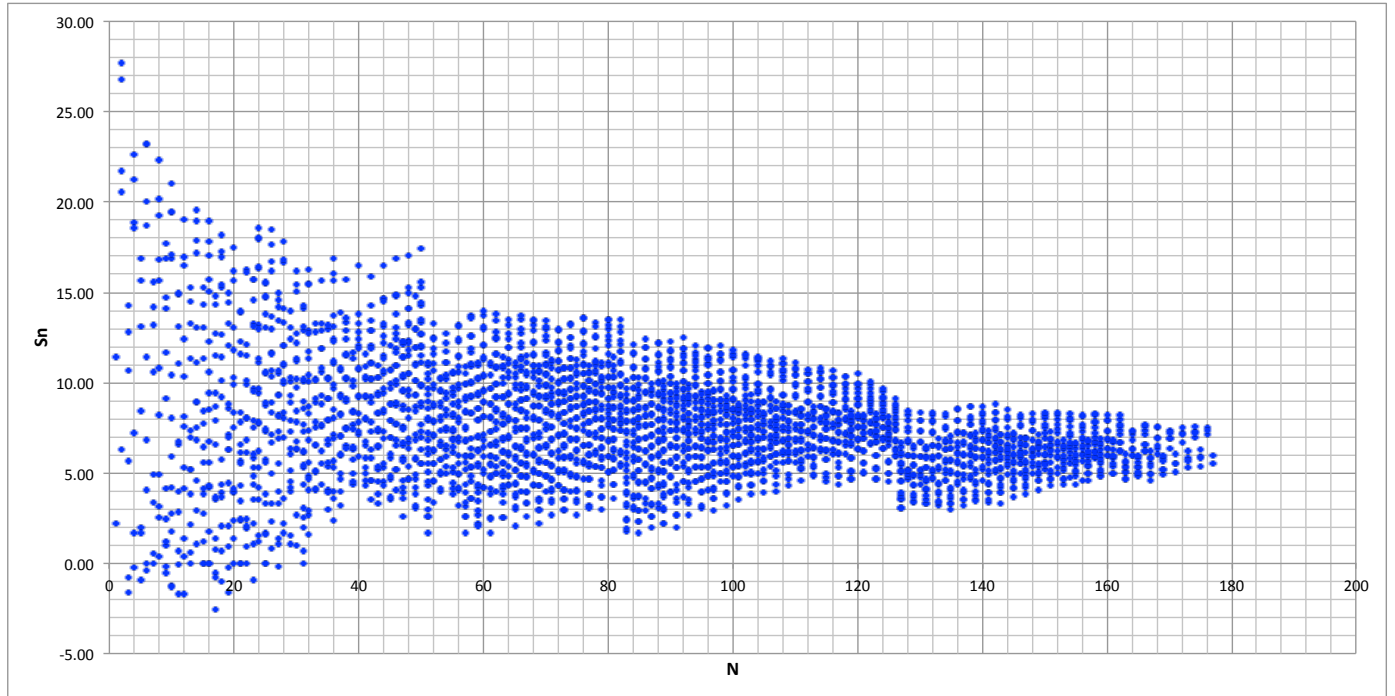


Fig. 6 Separation energy, S_n , versus N , for all known nuclides

In Fig 6, there are, indeed, discontinuities at $N=50$, $N=82$, and $N=126$. These discontinuities are considerably small, around 1 to 2 MeV, and they are minor compared to the overall spread, 30 MeV, of the experimental data for S_n that is seen in Fig. 6. Furthermore, these 1 to 2 MeV discontinuities are insignificant when compared to the overall binding energies (over 1800 MeV) for the larger nuclides. Even more problematic is that no such effect occurs with regard to magic numbers for the separation energy of a proton, S_p , and its absence is again a clear implication that nuclear shells are not a credible explanation.

There is no question that the small steps in S_n at $N=50$, 82 , and 126 exist. What is in question, however, is whether steps are the result of shells and magic numbers within the nucleus. With regards to the shell model, the term “magic” is very much a misnomer. Physicists know that magic is not really the reason or explanation for these nuclear behaviors as described above. Rather, the word “magic” is might be best explained as a shortened way of saying, “There is a phenomenon occurring at these numbers which we can not yet explain.” The shell model merely points out that at certain numbers of protons and neutrons, that there are certain phenomenon that exist in the behavior of nuclei. At some future time physicists may understand why those small steps occurs at $N=50$, 82 , and 125 for the separation energy S_n . However there is no question that such a future understanding would be an explanation of something other than magic.

Claim 5: There are deformations in the shape of the nuclei, as seen by the electric quadrupole moment, that occur when a nuclei is far away from a magic number.

Another claim of the shell model is that there are deformations in the shape of the nuclei when the nuclei is far away from a magic number. With regard to the concept of a spherical nuclides, it is claimed that these deformations from a spherical shape exist in small clusters within the nuclear chart, occurring far from the magic numbers. The experimental data for the electric nuclear quadrupole moments, which is directly correlated to the nuclear deformations, shows that this claim is not valid. If the shell model were correct, all of these quadrupole moments, seen in the Fig. 7, should be less than what is indicated by the blue line. (All data extracted from reference [4])

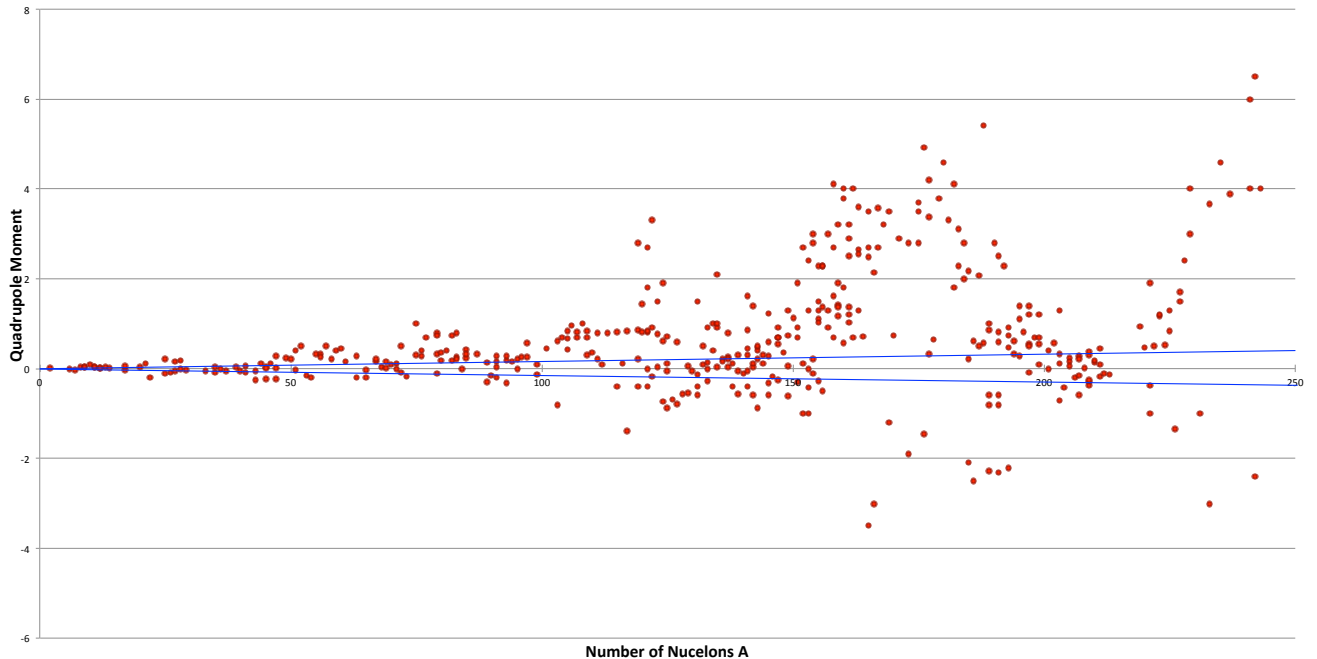


Fig. 7 Experimental quadrupole moment. The predicted deformation parameter of the shell model is shown by the blue lines.

As can be seen in Fig. 7, there are large deformations in the shape of the nuclides, which are unexplained by the shell model. This is true for the majority of nuclides, not just for small clusters far from the magic numbers. Hence this claim that the data for the electric nuclear quadrupole moment support the shell model is not supported by the updated empirical data. Most of the nuclides are far above the blue lines, the maximum quadrupole moment predicted by the shell model.

Discussion:

In conclusion, the experimental evidence for magic numbers is either relatively minor or it does not support the concept of nuclear shells. The updated experimental data for nuclides indicate that certain magic numbers may not be as cogent as previously assumed. This is an unanticipated result, but one which is difficult to deny when examining the updated experimental data. Thus, upon reexamination of updated data, the shell model appears to be less of a fundamental model to explain nuclear behavior than was previously believed in the 1950's. The shell model has seemed to failed the test of time in its ability to explain nuclear behavior.

References:

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