Question: Please explain the magnetic energy and how to calculate it for the magnetic dipoles of an up-to-down quark magnetic bond.

Answer:
The magnetic energy between two magnets is more complicated than the electric energy between two charges. This is because the magnetic energy has both vector and position dependence. Plus, the actual magnetic moments of the up and down quarks are not experimentally measured, but rather estimated theoretically from other parameters. This makes the determination of the magnetic energy not only more complicated, but somewhat uncertain.

The Magnetic Energy Between Two Magnets:
In order to determine the magnetic energy between two magnets, we must know the magnetic moments μ_1 and μ_2, of the two magnetic, as well as the magnetic field of $magnet_1$ at the location of $magnet_2$, symbolized as B_{12}. The magnetic energy, $U_{magnetic12}$, is the negative dot-product of the magnetic moment μ_2 with the field B_{12}, as shown in Eq. (1):

$Umagnetic12=-\mu2\cdot B12$ *Eq. 1*

For a collection of more than two magnets, the total magnetic energy is the double summation over all magnet pairs, as shown in Eq. 2:

$Umagnetic\ total=i=1n-1j=i+1n-\mu j\cdot Bij$ *Eq. 2*

where Bij is the vector magnetic field of the i^{th} magnet at the location of the j^{th} magnet.

The magnetic field is dependent on the vector orientation of the magnetic moment μ_i and the vector distance between the magnets, r_{ij}. The equation for Bij is a vector equation with an approximate, albeit somewhat complicated, $1/r^3$ dependence on distance. This is shown in Eq. 3:

$Bij=\mu04\pi3\mu i\cdot rijrij-rij2(\mu i)rij5$ *Eq. 3*

where μ_0 is the permeability constant of free space, $4\pi \times 10^{-7}$ in MKS units.

As a result of this approximate $1/r^3$ dependence, the magnetic field drops off quickly with distance. Thus, when the magnets are close to one another, this magnetic energy is quite large, but when the magnets are relatively distant from each other, the magnetic energy between them is minimal.

Combining equations 2 and 3, we see that the total magnetic energy of a distribution of magnets is shown in Eq 4.

Umagnetic total=i=1n−1j=i+1n−μ04π3μj•rjiμi•rji−rji2μi•μjrji5 *Eq. (4)*

Due to the vector properties of this energy, the strongest binding energy for two magnets is when the magnetic moments are stacked on top of one another, with the magnetic moments aligned and parallel, and as close as physically possible. This is shown in Fig. 1.

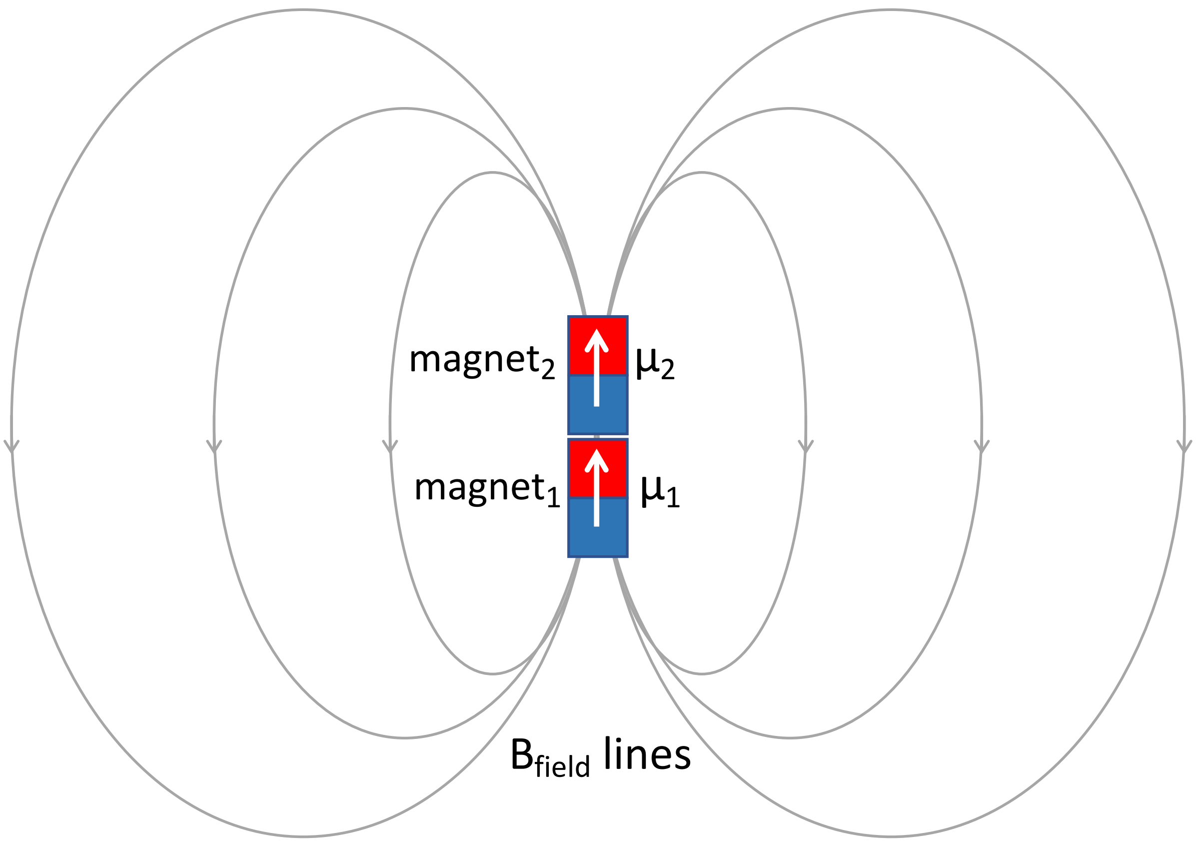

Fig. 1: The orientation with the strongest magnetic energy for two magnets: stacked, parallel, aligned, and as close to each other as possible.

Another way for two magnets to bond is a side-by-side bond, with the magnetic moments anti-parallel and the magnets oriented side-by-side, and as a close as physically possible. This binding energy is not as strong as the stacked magnetic bond. The side-by-side magnetic bond is shown in Fig. 2.

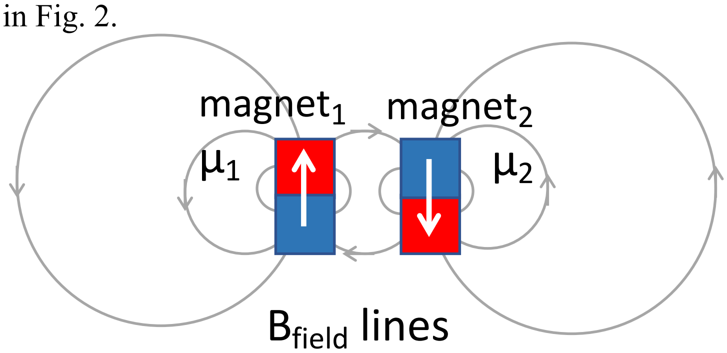

Fig. 2: A weaker orientation for the magnetic energy of two magnets: side-by-side, magnetic moments anti-parallel, and as close to each other as possible.

An angled bond, in between a stacked and a side-by-side bond, will give intermediate results, and the vector calculation is the same, using the appropriate vector components of μ, r, and B.

Calculating the Magnetic Energy of Quarks:
From quantum field theory, we know that quarks behave like point-like Dirac particles, each having their own inherent magnetic moment. In other words, similar to that of an electron, the magnetic moment of a quarks is a point-like magnetic moment, not caused by the current loop of spinning charge. This means in Fig. 1 and 2, that the magnets would be infinitesimally small in their physical dimensions.

The values of the magnetic moments for an up quark and down quark are not experimentally measured, but rather they are derived theoretically from other measurements, and estimated to be μ_{up}=+1.85 and μ_{down}=-0.97, in units of the nuclear magneton, μ_n, where the value of the nuclear magneton is 5.05 x 10^{-27} in MKS units. The radius of a proton is 0.842 x 10^{-15}, and the diameter is 1.684 x 10^{-15} meters. As an initial estimate, we use the assumption that the distance between two internucleon quarks can be 1/10 of the proton diameter; thus the estimated distance between the two internucleon quarks for this calculation is 1.684 x 10^{-16} meters.

If the magnetic bond is between one up and one down quark, in a side-by-side magnetic bond, we can simplify Equation 4, as shown in Eq 5:

$Umagnetic\ up\ to\ down=-\mu 04\pi 3\mu 2\bullet r21\mu 1\bullet r21-r212\mu 1\bullet \mu j2r215$ *Eq. 5*

For a side-to-side bond we can simplify by noting that for both magnets, the μ r dot product is zero. This is shown below in Eq. 6.

$Umagnetic\ up\ to\ down=-\mu 04\pi -r212\mu 1\cdot \mu 2r215$ *Eq. 6*

Simplifying further:
$Umagnetic\ up\ to\ down=-\mu 04\pi \mu 1\cdot \mu 2r213$ *Eq. 5*

Using the numbers for the magnetic moments, distance between the magnets, and the constants, we get the value, shown in calculation 1:

$Umagnetic\ up\ to\ down=(10-7)(0.97)(1.85)(5.05\times 10-27)2(1.68\times 10-16)3$

$Umagnetic\ up\ to\ down=9.62\times 10-13 joules=5.98\ MeV$

This is for the weaker side-by-side magnetic bond, at 1/10th the diameter of the proton; it is the magnetic energy only, not including the electric energy.

The actual internucleon quark-to-quark distance is about 1/8th the diameter of the proton. Within a nucleus, the energy of a magnetic bond depends on the angle and orientation of the bond. This gives the electromagnetic bond a range rather than a single value. For a quark-to-quark distance of about 1/8th the proton diameter, the range is from 4.50 MeV to 7.49 MeV per bond, depending on the vector orientation of the magnetic bond. Thus all electromagnetic bonds within a nucleus are not equal in value.

Clarifying the Confusion about Binding Energy

The binding energy that occurs as a result of an electromagnetic bond will put the overall configuration of nucleons into a lower energy state than the energy of the individual isolated constituent protons and neutrons. This is true for any kind of bond, be it electromagnetic, chromodynamic, or whatever type of bond it may be. When particles are bonded together, they are in a lower energy state than when they are isolated.

For a nucleus, the binding energy is the difference of the total energy of the bound nuclide compared to the total energy of the isolated constituent protons and neutrons. The binding energy is defined to be positive; however, this energy is subtracted from the energy of the unbound constituent parts. Thus, the energy and the mass of the bound nuclide are both lower than the energy and the mass of the constituent parts.

Since the binding energy (or binding mass) is subtracted from the energy of the individual protons and neutrons, the resulting energy and mass of the bound nuclide are both lower than that of its constituents. This lowering of energy and mass is a result that is very often misunderstood and commonly miscalculated--wherein it is erroneously added rather than subtracted. The binding energy is not added to the mass of the constituent particles, rather it is subtracted from them.

Thus, because this is so often misunderstood, this point bears repeating. The binding energy is subtracted from the energy of the individual protons and neutrons, and this results in an overall lower energy of the nuclide. Similarly, the binding mass is subtracted from the mass of the individual protons and neutrons, and this results in an overall lower mass of the nuclide. There is a decrease in the overall total energy and a decrease in the mass of the bound particle. The mass of the bound nuclide is less than the mass of the unbound constituent particles. The stronger the binding energy of a nucleus, the lower the overall energy of the nuclide.

Here are some references for more information about magnetic energy and the quark magnetic moments:

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