

A one parameter semi-empirical binding energy formula, based on quark flavor and quantum considerations

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Abstract

A one-parameter semi-empirical formula has been ascertained that can produce results similar to the five-parameter Weizsäcker formula, in terms of being able to predict nuclear binding energies to within a few percent. This one-parameter semi-empirical formula is similar to the residual chromodynamic model, in that the force binding the nucleons together in a nucleus is an internucleon quark-to-quark bond. However, this one-parameter semi-empirical formula is based upon the quark flavor rather than the quark color. The difference in the predicted binding energy between this flavor-based one-parameter formula and the actual binding energy is only a few percent difference. This result strongly suggests that the internucleon quark-to-quark bond described in the residual chromodynamic model has a stronger dependence on the quark flavor than previously presumed.

Keywords: Nuclear force, quark flavor, nuclear binding energy.

1. Introduction

The five parameter semi-empirical Weizsäcker formula [1] used in the liquid drop model of the nuclear force is the currently accepted model for estimating the experimental binding energy of a nuclide. The Weizsäcker formula is able to estimate the binding energy to within a few percent, using five empirically selected parameters and one logic condition.

However, the nuclear model most accepted for its association with the Standard Model is not the liquid drop model, but rather the residual chromodynamic (RCD) model. This RCD model claims that the force binding the nucleons together in a nucleus is a residual of the quantum chromodynamic (QCD) force. However, in a comparison of the QCD force versus the RCD force, it is seen that the two forces have different orders of magnitudes of strength, different mediating particles (gluons versus pions), different ranges (long range versus short range), and different regions of existence (inside versus outside the nucleon).

The QCD model states that a very strong bond is formed inside the nucleon between the quarks of differing colors [2]. The RCD model claims that there is a residual of this QCD force that exists outside the nucleon. The theoretical model of the RCD force is related to the Standard Model, due to its recognition of quarks within the nucleus, and it posits that the force for binding the nucleons together in a nucleus is due to an internucleon attractive color-related quark-to-quark force. In the RCD model, nucleons have three valence quarks—each having a different color of either red, green, or blue. (The colors of the quarks are a quantum attribute, unrelated to actual physical color.) This RCD force is an attractive force between quarks of different colors, forming a bond between the internucleon quarks, and thereby forming a bond between the associated nucleons.

Along with the quantum attribute of color, there is also another quantum attribute of quarks called “flavor”. From QCD theory, we know there are six different flavors of quarks: up, down, strange, charm, top, and bottom. Of these six different flavors, only two flavors are found in the stable matter of neutrons and protons: the up and down flavors [3]. (The terms up and down are a quantum attribute and do not imply any specific orientation with regard to spatial direction.) An up quark has an electric charge that is $+2/3$ the charge of a proton, and it also contains a magnetic moment, which is parallel to of the spin of the nuclide. The color of an up quark can be either red, green, or blue. A down quark has an electric charge that is $-1/3$ the charge of a proton, and it contains a magnetic moment, which is anti-parallel to of the spin of the nuclide. Similar to the up quark, the color of a down quark can be either red, green, or blue. The magnetic moment of an up quark is estimated to be $+1.85$ nuclear magnetons and the magnetic moment of a down quark is estimated to be -0.97 nuclear magnetons.

The electric charges of the proton and neutron are completely contained within the quarks. The proton is comprised of two up quarks and one down quark, giving it a net charge of one ($2/3 + 2/3 - 1/3 = 1$). The neutron is comprised of one up quark and two down quarks, giving it a net charge of zero ($2/3 - 1/3 - 1/3 = 0$).

2. The RCD Model of the Nuclear Force

The RCD model is the currently-accepted model of the nuclear force; however, it is extremely difficult to calculate or simulate with a computer, due to the complex nature of this force. As a result, the RCD model of the nuclear force is unable to duplicate the experimental binding energy curve, due to the extreme difficulty of simulating the multi-body interactions of the three color charges [4, 5]. Also, the QCD force between quarks inside the nucleons is very strong compared to the RCD force outside the nucleons. This disproportionate ratio of strength makes it difficult to find a converging solution for the complicated mathematical calculations. For nuclides with a small number of nucleons, the problem can be solved only with state-of-the-art powerful computers, done by putting each of the quarks into a four-dimensional lattice of discrete points: three dimensions of space and one dimension of time. This method is known as lattice QCD. For the computer simulations in lattice QCD, this method iterates the position of each quark by assigning an x, y, z, and t position to it, and calculating the resulting forces on each quark. As their positions change, due to these forces, this iterative procedure is repeated, until a converging solution is found. When a converging solution is found, then these calculations are able to determine the binding energy of the nuclide in question. These computer simulations are done through extremely complex mathematical models, often using

Monte-Carlo simulations [6]. Because of these computational difficulties, the binding energies of only the smallest nuclides have been simulated, with generally poor results. Due to these complications, this RCD model is not used as a standard nuclear physics tool [5]. Thus, the RCD model remains largely unverified when testing its binding energy predictions against experimental data.

The RCD model assumes that the quantum quark color exists a relatively short distance outside the nucleon, making this force a short range force. If two nucleons are close enough, this RCD force forms an internucleon quark-to-quark color-based bond, binding the nucleons together, as shown in the illustrative conceptual representation, in Fig. 1. In Fig. 1, the black lines represent the chromodynamic force inside the nucleon, and the dotted line represents the weaker residual chromodynamic force between two nucleons. (Quarks are considered to be nearly point-like particles. Thus, this drawing is not meant to be a scaled representation of the relative sizes of the quarks, rather it is meant for illustrative and conceptual purposes only.) The RCD bond is between any two quarks of different colors, such as between a red and blue, a green and red, or a blue and green. This internucleon bond between the quarks forms a bond between two nucleons.

Note that any bond between the different colors is also inherently a bond between the flavors of the quarks. Thus, the quantum assumptions that are made in the RCD model, regarding the internucleon quark-to-quark bonds between the colors of quarks, are inherently applicable to the formation of an internucleon bond between the flavors of up and down quarks. Fig. 2 shows three possible bonds, all of which are allowed in the RCD model: a bond between two up quarks, between two down quarks, and between an up and a down quark. In the RCD model, as long as the quark-to-quark bond is between different colors, the up or down flavor of the quarks is irrelevant.

Although it is considered unimportant in the RCD model, the up or down flavors of the quarks do indeed cause an energy difference, among the three types of bonds shown in Fig. 2. If there is an internucleon bond between an up quark and another up quark, then the intrinsic electromagnetic force between these quarks is repulsive. Similarly for a bond between a down quark and another down quark, the bond has an electromagnetic component that is repulsive. Conversely, if there is an internucleon bond between an up quark and a down quark, then the intrinsic electromagnetic force between these quarks is attractive. Among the three types of bonds shown in Fig. 2, this intrinsic difference in the electromagnetic energy causes the two electromagnetically-repulsive bonds to be less probable and less stable, producing a situation wherein the up-to-down quark bond would be more prevalent in stable matter.

3. An Additional Constraint for Internucleon Quark-to-Quark Binding

The color attribute of the quarks contained within a nucleon do not determine whether or not the nucleon is a neutron or a proton; only the up or down flavor attribute of the quarks can distinguish the difference between a proton and a neutron. Therefore, an appropriate possibility to explore is an examination of an internucleon bond being formed *only* between an up and a down quark—meaning that the up-to-up quark bond and down-to-down quark bonds are not allowed. In other words, for a stable bond to form between two nucleons, the internucleon quark-to-quark bond must be between an up and a down quark (as well as being between two difference colors). When this reasonable constraint is made to the RCD model, a quick calculation of the number of allowed bonds can be made. By using the currently-accepted RCD concept of the internucleon quark-to-quark bond, and by applying this additional constraint—wherein bonds are only formed between up and down quarks—potentially significant results emerge.

For any given nuclide, the number of internucleon up-to-down quark pairs can be determined, based on how many up and down quarks each nuclide has. This calculation, shown in Eq. (1), is made for each nuclide.

$$\begin{aligned} \text{Number}_{\text{up quarks}} &= (Z \times 2) + (N \times 1) \\ \text{Number}_{\text{down quarks}} &= (Z \times 1) + (N \times 2) \\ \text{Number}_{\text{possible pairs}} &= \text{smaller of these two numbers.} \end{aligned} \quad (1)$$

For simplicity, it is assumed that every bonded pair of up-to-down quarks has the same bonding energy. (For this very simple calculation, any minor deviations in an internucleon up-to-down quark bond are ignored.) For this calculation, the equation for the calculated binding energy (CBE) is the number of internucleon up-to-down quark pairs times the constant binding energy per bonded pair, shown in Eq. (2). (Equation2 is for a non-quantum nuclide wherein no quantum properties have yet to be considered.)

$$\text{CBE} = (\text{Number}_{\text{u-d pairs}}) \times (E_{\text{constant per bond}}) \quad (2)$$

where: $\text{Number}_{\text{u-d pairs}}$ is the number of up to down quark pairs, and $E_{\text{constant per bond}}$ is the fixed energy per bonded pair.

For a representative sample of stable nuclides, this information is shown in Table 1. (For $A=5$ and $A=8$, there are no stable isobars, but these values of A are included in the table for continuity.) The following information is listed in the first 9 columns of Table 1.

1. The nuclide name
2. The number of nucleons, A
3. The number of protons, Z
4. The number of neutrons, N
5. The experimental binding energy (EBE) in MeV, obtained from reference [7].
6. The experimental binding energy per nucleon (EBE/ A)
7. The number of up quarks in the nuclide
8. The number of down quarks in the nuclide
9. The number of possible pairs between up and down quarks, not taking any quantum rules into consideration.

For this one-parameter flavor-based calculation, neither the type of bond nor the structure of the bond comes into consideration. Simply stated, this is a theory-independent calculation of the number of possible bonded pairs multiplied by a fixed binding energy per bond.

4. Quantum Considerations

A nucleus is a quantum object, and being so, certain quantum considerations apply. One known phenomenological feature of the nuclear force is the hard-core repulsion. The hard-core repulsion states that nucleons cannot overlap in their spatial location [8, 9]. If too many bonds are formed for either ${}^2\text{H}$, ${}^3\text{He}$, or ${}^4\text{He}$, then overlap occurs. This is illustrated in Fig. 3.

To prevent this overlap, hydrogen ${}^2\text{H}$ can have only one bond instead of two or three bonds. Similarly, helium ${}^3\text{He}$ can have only three bonds instead of four, and ${}^4\text{He}$ can only have 5 bonds. Three other stable nuclides—those with odd Z , odd N , and $Z=N$ —are also subject to this constraint: ${}^6\text{Li}$, ${}^{10}\text{B}$, and ${}^{14}\text{N}$. Specifically, the odd neutron and the odd proton cannot bond twice to each other, forming a doubly-bonded nucleon. Other stable nuclides are not affected by the application of this rule.

Quantum mechanics also states there can be no net electric dipole moment for a nucleus [10]. For this second quantum consideration, three more bonds must be subtracted from the number of bonds available, in order to remove the electric dipole moment in each of the three dimensions. Without stating any specific configuration for the nuclide, this reduction of bonds can be best understood from the fact that the electric charge distribution of the nuclide must not have a net asymmetry in electrical charge for any of the three spatial dimensions, x , y , or z . This is illustrated in Fig. 4.

Fig. 4 shows a randomly-shaped object with a dipole moment in the horizontal dimension. Also shown are the up-to-down quark bonds, dispersed throughout the object. In the left-most figure, there is a dipole moment, due to the imbalance of charged, indicated by the arrows. To remove this dipole moment, a bond must be broken, and the charges are rearranged to remove this imbalance. In the center figure, a bond is broken. In the right-most figure, the charges have been rearranged to give a balanced charge distribution, and the dipole moment has been removed in that horizontal dimension.

To prevent an electric dipole moment, the charge distribution must be balanced in each of the three spatial dimensions. Specifically for a distribution of charge wherein the up and down quarks are bonded, a bond is broken in each of these three dimensions in order to remove the dipole. Thus, this quantum requirement removes three of the previously-allowed bonds, and it applies to all stable nuclides—with the exception of the three smallest stable nuclides— ${}^2\text{H}$, ${}^3\text{He}$, and ${}^4\text{He}$ —that don't have enough bonds to break.

The inclusion of these two quantum rules is also shown in Table 1, in the columns 10 through 13. The first 9 columns of Table 2 are similar to the first 9 columns of Table 1. Also shown in Table 2 is the number of possible quantum bonds for each nuclide, taking into consideration the two above mentioned quantum rules. The last three columns of Table 1 show the quantum CBE, the CBE/A, and the percent difference of that calculated binding energy (CBE) as compared with the experimental binding energy (EBE).

As before, the calculated binding energy is the number of bonds multiplied by a fixed energy per bond. The energy per bond, in this example, is the one empirically-selected parameter. For this simple calculation, it is 6.000 MeV per bond. These data are plotted in Figs. 5 and 6. In Fig. 5, a representative sample of all of the stable nuclides are shown, going out to ${}^{204}\text{Pb}$. In Fig. 6, only the first 50 nuclides are plotted, to show the detail.

To reiterate, this is an easy and quick calculation, involving only a numerical count of the quantum-allowed internucleon up-to-down quark pairs. This calculation does not specify the configuration or arrangement of the nucleons. Nor does it describe the exact mechanism or forces within the bond. It is simply a count of the quantum-allowed up-to-down quark bonds. Other than being quantum, this calculation is theory independent.

5. Discussion

The remarkably good reproduction of the experimental data for these calculated results is impressive, especially considering that there is only one empirically-selected parameter for this calculation—the value of 6.000 MeV for the bond energy—instead of the five empirically-selected parameters for the Weizsäcker formula. This reproduction of the experimental data is especially impressive, considering that the currently-accepted RCD model of the nuclear force is unable to duplicate this curve. Furthermore, no large state-of-the-art computers are required to run extremely complicated calculations.

In terms of the possible mechanism or force for the internucleon quark-to-quark bond, the RCD force between the colors is one possibility. Another possible force might be a “flavor-force” between an up and a down quark. A third possible force for this bond becomes apparent when it is recalled that the up quark has an electric charge $+2/3$ the charge of a proton, the down quark has an electric charge of $-1/3$ the charge of a proton, and both quarks carry a magnetic moment. These electromagnetic properties of the up and down quarks create a strong attractive electromagnetic force between them; the strength of this electromagnetic force is dependent only on the minimum proximity between the up and down quarks engaged in the bond.

Historically, it was believed that the strength of the electromagnetic force had an upper limit, based on the misconceptions that protons were elementary particles and that they were homogeneously charged. However, when quarks, which contain the electric charge and magnetic moment for the nucleons, are taken into consideration, these misconceived notions are now known to be invalid. In the limit as the internucleon quark-to-quark distance approaches zero, the strength of the electromagnetic force approaches infinity; thus, this electromagnetic bond can be extremely strong.

The internuclear quark-to-quark bond may be a combination of both the electromagnetic force, a flavor force, and a color force of the quarks. However, the relative percentages of these three contributions are not postulated here. Regardless of the relative percentages, the electromagnetic component of this bond should not be ignored or disregarded. When any internucleon quark-to-quark bond is considered, the *electromagnetic component of this bond must be taken into full account*, and it should not be minimized as being unimportant or inconsequential.

Regarding the electromagnetic contribution of this internucleon flavor-based quark bonding, a more detailed analysis can easily be made by using the standard electromagnetic equations and quantum physics. A detailed analysis would include the electrical energy of all electric charges as they interact with each other. This calculation would inherently include the Coulomb energy of the net repulsive electric energies among the protons. The energy of the magnetic dipole moments, as they interact with each other, should also be included. Such a detailed electromagnetic analysis would include the variation of the magnetic energy due to the vector orientation of the

magnetic dipole moments of the quarks. As a result of this variation of magnetic energy within the bond, there would be a consequent variation in the energy of the internucleon up-to-down quark bonds. Furthermore, the kinetic energy of the quantum spin of the nuclide should also be included in this more detailed binding energy calculation.

6. Conclusion

A one-parameter semi-empirical formula of the internucleon flavor-based quark bond has been ascertained, giving remarkably good results in duplicating the nuclear binding energy curve. This calculation does not require large expensive state-of-the-art computers. While the currently-accepted RCD model has difficulty simulating the binding energy for only a few nucleons, this calculation has no such restrictions, and can easily simulate even the very largest nuclides in the nuclear table.

For this one-parameter formula, the resulting differences between the experimental and calculated binding energies for the stable nuclides are only a few percent. Also, due to the inherent similarities of this concept to the residual chromodynamic force model—with its internucleon color-related quark-to-quark bond—the existence of an internucleon flavor-related quark-to-quark bond is already considered a plausible and accepted concept. (Thus, in reply to any potential objections of an atomic nucleus actually having an internal structure, it should be noted that this concept is already part of the currently-accepted RCD model.)

The remarkably good reproduction of the experimental binding energy curve, with only one empirically-selected variable, strongly indicates that the internucleon flavor-based quark bond is a substantial and significant part of the force that binds the nucleons together in an atomic nucleus. One other implication of the results of this one-parameter semi-empirical formula is that a significant portion of the nuclear force may be electromagnetic. Although this may be an unexpected conclusion, it is, nonetheless, feasible—especially when the electromagnetic attraction of the up-to-down quarks is considered. Historically, when only the misconceived homogeneously-charged protons are considered, then these hypothetical protons could not bond to other each other electromagnetically, and the concept that the nuclear force could be electromagnetic, even partially, was deemed impossible. However, with the understanding that the electrical charges of the internucleon up and down quarks are able to attract and bond to each other, then such a concept—that a significant portion of the nucleon-to-nucleon bond might be electromagnetic—is possible.

This further suggests that the concept introduced in this paper—of internucleon up-to-down quark bonding—should not be discarded simply because it is a slightly different refinement of the currently-accepted residual chromodynamic force model. Rather, the concept of internucleon flavor-based quark bonding should be seriously considered, with further theoretical investigation. This result strongly suggests that the internucleon quark-to-quark bond described in the residual chromodynamic force model has a much stronger dependence on the quark flavor than previously presumed.

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